Optimal Project Portfolio Management

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Contents

1 Executive Summary 1

2 Introduction 2
  2.1 Background . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2
  2.2 Constraints . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2
  2.3 Methodology . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3

3 Modeling 3
  3.1 Decision Variables . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3
  3.2 Objective Function . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4
  3.3 Constraint Formulation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4

4 Analysis 7
  4.1 Solving the Linear Program . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 7
  4.2 Exploration: Preemption . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 8
  4.3 Exploration: Budget Variation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 9
  4.4 Exploration: Budget Variation with Preemption . . . . . . . . . . . . . . . . . . . . 10
  4.5 Summary . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 11

5 Recommendations 12

6 Appendix 13
  6.1 AMPL Data Input . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 13
  6.2 AMPL Code . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 14
1 Executive Summary

CalInvestor is a Berkeley-based investment company that invests in promising and innovative technology projects for profit. Unlike its competitors, CalInvestor receives a great number of high-quality project plans for the next three months. Completing a multitude of projects will generate huge amounts of profit for the company.

Investment firms are typically subject to short-term investment budget and the constraint that several projects depend on each other. Investment in a technology project requires a great amount of money to put in, sometimes extremely expensive for very innovative technology. At the same time, developing a novel product may require completing dozens of projects in advance, possibly spanning several phases. As a result, many immature investors are unable to efficiently allocate the resources to invest in brand new technology projects effectively.

The analysis team provides a complete and payoff-maximum solution to generate the most efficient and effective project-scheduling portfolio. It is suggested that those 12 potential projects start and end according to Table 2 on page 8. A detailed project-scheduling portfolio can be seen in Figure 1 on page 8. Following the diagram, CalInvestor can achieve the maximum net profit, satisfying all necessary constraints. This optimal solution is therefore the best solution - achieving the highest net profit - among all possible scheduling portfolio under those limitations.

At the same time, the team also analyzes all possible variation in changeable constraints. Budget limit for each period can be raised to 7 million dollars, instead of the current 5 million dollars. Projects may also pause and start over to fit the budget limit and other constraints. It has been shown in the analysis below that CalInvestor can complete more projects within the period, applying either of those two changes. Applying both changes can even make the most amount of projects. It is very clear that completion of more projects will boost the total net profit.

Although with those variations, CalInvestor can generate more net profit, the optimal solution provided include neither modification. When considering the rate of return, the ratio of total net profit to total cost, the team finds out that this solution has the highest expected rate of return. In other words, it implies this is the most effective investment among all choices. Instead of putting more money in the project innovation and achieve a lower rate of return, investing a decent amount of money can not only generate a higher relative net profit, but also provide enough support for other financial activities in the company. Therefore, the final suggestion is made based on the complete discussion on all possible improvement and analysis.

As a financial summary, the investment team will seek $120.8 million dollars in total for investment funding. These funds will be used to finance the development and acquisition of current technology projects, and creation of innovative products for sales. At the conclusion of those projects, the company expects to achieve a net profit of $104.2 million dollars with an expected rate of return of 86%. This profit will make the company be in the position of a pioneer in technology investment industry.
2 Introduction

2.1 Background

Over the next three years, CalInvestor will be working on 12 potential projects. It is a good opportunity for the company to create new products by completing those projects spanning several periods. Some of those projects even produce innovative technologies that are new in the market and will result in significant revenue. Therefore, a good operating strategy is to complete as many projects as possible.

However, budget limit and human resource are two important factors restricting the completion of those projects. The company is now restricted to use a fixed amount of budget every month for the research and development (R&D) on those projects. Meanwhile, people focusing on a certain project need to concentrate on it until it is completed. In addition, technology development must follow a certain procedure, completing all necessary projects in an ordered sequence. Given all those constraints, optimizing CallInvestor’s project portfolio for the next three years is critical. Those projects must be managed efficiently to achieve the maximum profit. Therefore, the goal of the report is to find the most efficient arrangement of those projects for each month, under proper sequences and fixed budgets.

2.2 Constraints

Although completion of as many projects as possible gains profit the most, based on the background information above, those unavoidable requirements and constraints must also be taken into account. A detailed summary of those constraints is listed below.

- The portfolio will be measured in monthly period, hence 36 periods in total.
- There are 12 potential projects. Operational cost, duration, and total profit for each project are shown in the table below.

<table>
<thead>
<tr>
<th>Project</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost $c_i$ (millions / month)</td>
<td>1.1</td>
<td>1.2</td>
<td>1.3</td>
<td>1.4</td>
<td>2.5</td>
<td>3.6</td>
<td>1.7</td>
<td>1.8</td>
<td>1.9</td>
<td>2</td>
<td>1.8</td>
<td>1.9</td>
</tr>
<tr>
<td>Duration $d_i$ (months)</td>
<td>7</td>
<td>6</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>6</td>
<td>7</td>
<td>12</td>
<td>10</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Profit $p_i$ (millions)</td>
<td>2</td>
<td>8</td>
<td>15</td>
<td>4</td>
<td>10</td>
<td>11</td>
<td>16</td>
<td>23</td>
<td>20</td>
<td>50</td>
<td>60</td>
<td>70</td>
</tr>
</tbody>
</table>

Table 1: Operational cost, duration, and total profit for each project

- The budget constraint is set to be 5 million dollars per month.
- Due to government regulations, no more than 3 new projects can start in a same month.
- There exists certain dependence among some projects above. Some projects must be completed before a new project starts. Some projects may compete resources, which implies at most one of those projects at a time. A detailed requirement is summarized as followings:
  1. Project 3 must be completed before project 9 starts.
  2. Projects 1 and 2 must be completed before project 10 starts.
  3. Project 1 must be completed before project 11 starts.
4. Project 2 must be completed before project 12 starts.
5. Project 4 or 5 must be completed before project 10 starts.
6. Project 6, 7 or 8 must be completed before project 11 starts.
7. Project 9 or 11 must be completed before project 12 starts.
8. Project 1, 3 and 8 cannot overlap.
9. Project 2 and 4 cannot overlap.
10. There must be a gap of at least 1 month between project 5 and 6.

2.3 Methodology

The technique using in this report is mathematical programming. A mathematical model, whose requirements are represented by linear constraints, will be built to achieve the best outcome – maximum profit. The model can output the most efficient timing of launching each project. All of the modeling and analysis below will use integer programming technique, a very common mathematical programming tool used in the industry.

3 Modeling

3.1 Decision Variables

Decision variable is a quantity that can be manipulated in search of values that produce the optimal value. In the context of the model, three necessary decision variables are defined as followings:

1. $S_{ij}$ denotes whether project $i$ initially starts in month $j$.
   - It is a binary indicator, taking value of 1 when project $i$ starts in month $j$ or 0 otherwise.
   - Since each project can only start once, for each project there is at most one indicator taking value of 1 in all 36 periods. Taking an example for project 1, there should be at most one indicator that $S_{1j} = 1$, $j \in [1, 36]$. At least 35 indicators for project 1 should then be equal to 0.
   - $\sum_{j=1}^{36} S_{ij}$ indicates whether or not project $i$ ever starts over the total 36 periods. It can be treated as a binary variable, thus defined as project starting indicator in this modeling. If project $i$ never starts during 36 periods, the sum of all starting indicators for that project should be equal to 0. If it gets started, project starting indicator will take value of 1. $\sum_{j=1}^{k} S_{ij}$ is similar, indicating whether or not project $i$ starts within the first $k$ periods.
   - $\sum_{j=1}^{36} S_{ij} \cdot j$ gives the numerical number of the starting month for project $i$. Reasoning is based on the previous two bullet points. If project $i$ never starts during 36 periods, it will output 0.

2. $E_{ij}$ denotes whether project $i$ completely ends in month $j$.
   - It is a binary indicator, taking value of 1 when project $i$ ends in month $j$ or 0 otherwise.
• Similar to starting indicator, there is at most one ending indicator taking value of 1 in all 36 periods. At least 35 indicators for each project should then be equal to 0.

• \( \sum_{j=1}^{36} E_{ij} \) indicates whether or not project \( i \) ever ends over the total 36 periods. It can also be treated as a binary variable, thus defined as project ending indicator in this modeling. \( \sum_{j=1}^{k} E_{ij} \) indicates whether or not project \( i \) starts within the first \( k \) periods.

• \( \sum_{j=1}^{36} E_{ij} \cdot j \) gives the numerical number of the ending month for project \( i \).

3. \( O_{ij} \) denotes whether project \( i \) is currently operating in month \( j \).

• It is a binary indicator, taking value of 1 for all periods in its duration, and 0 for the rest.

• Project \( i \) should completely operate throughout all its duration, including its beginning month and ending month.

3.2 Objective Function

Based on the description above, the goal is to maximize the revenue after 36 periods of running those projects. The revenue for the portfolio as a whole should be equal to its total profit minus its total cost. A formal function in mathematical notation is as following:

\[
\max \sum_{i=1}^{12} \left[ \left( \sum_{j=1}^{36} S_{ij} \right) \cdot (p_i - c_i \cdot d_i) \right]
\]

Here is the interpretation of the objective function above. The total revenue is the sum of the revenue of 12 individual projects. If the project \( i \) gets started during the 36 periods, the revenue for it is its profit \( p_i \) minus its total cost. Its total cost is expected to be the cost per month \( c_i \) times the duration \( d_i \) (Note: Refer \( p_i, c_i \) and \( d_i \) to Table 1 on page 2). Since there is at most one starting indicator \( S_{ij} \) for project \( i \) taking value of 1, the sum of indicators for all 36 months can only be either 0 or 1. If the value is 1, project \( i \) is running during the period and its profit will contribute to the total revenue. Otherwise, a value of 0 implies not running project \( i \) during the periods, so that 0 will be added to the total revenue.

3.3 Constraint Formulation

The heart of this model building is clearly identifying those constraints. There are two types of constraints in the model: constraints from the property of the decision variables, and constraints shown in section 2.2. Constraints 1 to 8 shown below are from the property and relationship of those three decision variables defined in section 3.1.

\[ S_{ij}, E_{ij}, O_{ij} \geq 0, \text{ and binary} \]  

(1)

Constraint 1 states that all of those three decision variables should be nonnegative based on its definition. Especially those are binary variables which can only take value of 0 and 1.

\[ \sum_{j=1}^{36} S_{ij} \leq 1, \forall i \in [1, 12] \]  

(2)
\[
\sum_{j=1}^{36} E_{ij} \leq 1, \forall i \in [1, 12] \quad (3)
\]

Constraint 2 and 3 translate the property of starting and ending indicators in Section 3.1. For project \(i\), there is at most one \(S_{ij}\) (or \(E_{ij}\)) variable taking value of 1 over all 36 periods. No project can start or end more than once.

\[
\sum_{j=1}^{36} O_{ij} = d_i \cdot \sum_{j=1}^{36} S_{ij}, \forall i \in [1, 12] \quad (4)
\]

Constraint 4 implies that the total operating time for project \(i\) should be equal to its duration \(d_i\) in Table 1. If project \(i\) never gets started, both sides of the equation above should be equal to 0. It leads to put the project starting indicator on the right-hand side of the equation.

\[
\sum_{j=1}^{36} S_{ij} = \sum_{j=1}^{36} E_{ij}, \forall i \in [1, 12] \quad (5)
\]

Constraint 5 denotes the relationship between \(S_{ij}\) and \(E_{ij}\). In order to maximize total revenue, projects must be completed for profit if it gets started. Hence there are only two circumstances for any project \(i\) during the periods: complete the project before the last period, or never start the project. Therefore project starting indicator should always be equal to project ending indicator.

\[
\sum_{j=1}^{36} E_{ij} \cdot j - \sum_{j=1}^{36} S_{ij} \cdot j = (d_i - 1) \cdot \sum_{j=1}^{36} S_{ij}, \forall i \in [1, 12] \quad (6)
\]

Constraint 6 shows the relationship among \(S_{ij}\), \(E_{ij}\), and \(d_i\). If project \(i\) gets started, it must be continuously running in the next few periods until it is completed. The difference of numerical number of starting month and ending month exactly captures the duration of the project (from a counting perspective, the difference is actually the duration minus one). Similar to constraint 4, if project \(i\) never gets started, both sides of the equation above should be equal to 0. Project starting indicator is therefore multiplied on the right-hand side of the equation.

\[
\sum_{j=1}^{k} S_{ij} \geq O_{ik}, \forall i \in [1, 12], \forall k \in [1, 36] \quad (7)
\]

\[
\sum_{j=k}^{36} E_{ij} \geq O_{ik}, \forall i \in [1, 12], \forall k \in [1, 36] \quad (8)
\]

Constraints 7 and 8 set up the boundary for the operating indicator \(O_{ij}\). Operating indicators cannot take value of 1 before the project starts, or after the project ends. Therefore, \(\sum_{j=1}^{k} S_{ij}\), indicating whether or not project \(i\) starts within the first \(k\) periods, restricts the operating indicator at period \(k\). Similarly, the partial sum of ending indicators also bounds the operating indicator.

Constraints 9 to 23 shown below are from budget limit and project sequence requirement described in section 2.2.
\[
\sum_{i=1}^{12} O_{ij} \cdot c_i \leq 5, \forall j \in [1, 36] \tag{9}
\]

Constraint 9 corresponds to the budget limit per period. For each period \(j\), the total cost of all currently operating projects should be less than 5 million dollars.

\[
\sum_{i=1}^{12} S_{ij} \leq 3, \forall j \in [1, 36] \tag{10}
\]

Constraint 10 corresponds to at most three newly starting projects per period. For each month \(j\), the sum of all starting indicators \(S_{ij}\) should be less than 3.

\[
\sum_{j=1}^{k} E_{3j} \geq \sum_{j=1}^{k+1} S_{9j}, \forall k \in [1, 35] \tag{11}
\]

Constraint 11 corresponds to the requirement that project 3 must be completed before project 9 starts. If project 3 ends in month \(j\), then project 9 must start no earlier than month \(j + 1\). The left-hand side of the inequality denotes whether or not project 3 ends within the first \(k\) periods. The right-hand side of the inequality denotes whether or not project 9 starts within the first \(k + 1\) periods. The left-hand side restricts the right-hand side for all 36 periods.

\[
\sum_{j=1}^{k} E_{1j} \geq \sum_{j=1}^{k+1} S_{10j}, \forall k \in [1, 35] \tag{12}
\]

\[
\sum_{j=1}^{k} E_{2j} \geq \sum_{j=1}^{k+1} S_{10j}, \forall k \in [1, 35] \tag{13}
\]

\[
\sum_{j=1}^{k} E_{1j} \geq \sum_{j=1}^{k+1} S_{11j}, \forall k \in [1, 35] \tag{14}
\]

\[
\sum_{j=1}^{k} E_{2j} \geq \sum_{j=1}^{k+1} S_{12j}, \forall k \in [1, 35] \tag{15}
\]

Constraints 12 and 13 correspond to the requirement that projects 1 and 2 must be completed before project 10 starts. Constraint 14 corresponds to the requirement that project 1 must be completed before project 11 starts. Constraint 15 corresponds to the requirement that project 2 must be completed before project 12 starts. All reasoning is similar to constraint 11.

\[
\sum_{j=1}^{k} E_{4j} + \sum_{j=1}^{k} E_{5j} \geq \sum_{j=1}^{k+1} S_{10j}, \forall k \in [1, 35] \tag{16}
\]

Constraint 16 corresponds to the requirement that project 4 or 5 must be completed before project 10. The reasoning is first based on constraint 11. If either project 4 or 5 ends in month \(j\), then project 9 must start no earlier than month \(j + 1\). The ”either-or” relationship is denoted by the addition above in the inequality, since once one project is completed, project 10 can get started too.
The sum of those two parts indicates whether or not at least one of project 4 and 5 has started. The inequality should hold for all month \( j \), except the last one.

\[
\sum_{j=1}^{k} E_{6j} + \sum_{j=1}^{k} E_{7j} + \sum_{j=1}^{k} E_{8j} \geq \sum_{j=1}^{k+1} S_{11j}, \forall k \in [1, 35] \quad (17)
\]

\[
\sum_{j=1}^{k} E_{9j} + \sum_{j=1}^{k} E_{11j} \geq \sum_{j=1}^{k+1} S_{12j}, \forall k \in [1, 35] \quad (18)
\]

Constraint 17 corresponds to the requirement that project 6, 7 or 8 must be completed before project 11 starts. Constraint 18 corresponds to the requirement that project 9 or 11 must be completed before project 12 starts. Both reasoning is similar to constraint 16.

\[
O_{1j} + O_{3j} + O_{8j} \leq 1, \forall j \in [1, 36] \quad (19)
\]

Constraint 19 corresponds to the requirement that project 1, 3 and 8 cannot overlap. In other words, project 1, 3 and 8 cannot be operating together in any period. There is at most one indicator taking value of 1 in each period among those three operating indicators \( O_{ij} \), one for each project.

\[
O_{2j} + O_{4j} \leq 1, \forall j \in [1, 36] \quad (20)
\]

Constraint 20 corresponds to the requirement that project 2 and 4 cannot overlap. The reasoning is similar to constraint 19.

\[
O_{5j} + O_{6j} \leq 1, \forall j \in [1, 36] \quad (21)
\]

\[
O_{5j} + O_{6j+1} \leq 1, \forall j \in [1, 35] \quad (22)
\]

\[
O_{5j} + O_{6j-1} \leq 1, \forall j \in [2, 36] \quad (23)
\]

Constraints 21, 22 and 23 correspond to the requirement that there must be a gap of at least 1 month between project 5 and 6. Similar to constraint 19, project 5 and project 6 cannot overlap for sure. Hence, there are only two circumstances: project 5 followed by project 6, or project 6 followed by project 5. In the first case, it is necessary to restrict that project 5’s starting month cannot overlap with the month after project 6’s ending month. Therefore, project 5’s operating month cannot overlap with the month immediately after project 6’s operating month. The same reasoning also applies for the second case.

### 4 Analysis

#### 4.1 Solving the Linear Program

Once the model is built, solving the linear program will provide the optimal solution for the project management. Code in AMPL (a modeling language for mathematical programming) has been written for the model above. Code is attached in the Section 6. Thanks to UC Berkeley computing resources, the result, after running the code, is shown in Table 2.
<table>
<thead>
<tr>
<th>Project</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting Month</td>
<td>1</td>
<td>10</td>
<td>11</td>
<td>1</td>
<td>NA</td>
<td>NA</td>
<td>2</td>
<td>NA</td>
<td>NA</td>
<td>25</td>
<td>11</td>
<td>25</td>
</tr>
<tr>
<td>Ending Month</td>
<td>7</td>
<td>15</td>
<td>20</td>
<td>9</td>
<td>NA</td>
<td>NA</td>
<td>8</td>
<td>NA</td>
<td>NA</td>
<td>36</td>
<td>22</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 2: Optimal solution for the original linear program

Entry with "NA" in the field denotes the project never starts within 36 periods. If those projects are managed to start and end according to Table 2, there are 8 projects running during the whole period. CalInvestor can generate a net revenue of 104.2 millions dollars within 3 years, at maximum. A detailed project portfolio management schedule is plotted below. Marked squares show the current project operating during certain months.

![Optimal project portfolio for the original linear program](image)

According to the introduction in Section 2.1, budget limit and human resource are two important factors impeding the project management. In the next three subsections, slight variations on those two factors will be explored, in order to see if a larger net revenue can be generated.

4.2 Exploration: Preemption

In the previous analysis, once a project is launched, the project must be kept running until fully completed. Human resources is major reason restricting the project operation. However, there is another possible approach to make the project management more efficient. Preemption is the technique that allows project to pause and start off later. By introducing more flexibility in the schedule, preempted project can then start off at where it is paused.

Slight modification to the original model is sufficient for the preemption. The decision variables are still those three, where $S_{ij}$ denotes the initial starting indicator, $E_{ij}$ denotes the final starting indicator, and $O_{ij}$ is still the the operating indicator. Then the only constraint to be modified is constraint 6. Instead of making the difference of starting and ending date be the duration of the project, the difference in preemption needs to be greater than its duration. The modified constraint
is shown below.

$$\sum_{j=1}^{36} E_{ij} \cdot j - \sum_{j=1}^{36} S_{ij} \cdot j \geq (d_i - 1) \cdot \sum_{j=1}^{36} S_{ij}, \forall i \in [1, 12]$$

(24)

AMPL code for preemption also needs to be modified to reflect the change. The change is commented in the Section 6 Appendix. After running the modified code, the optimal solution is shown in Table 3.

<table>
<thead>
<tr>
<th>Project</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting Month</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>2</td>
<td>NA</td>
<td>NA</td>
<td>1</td>
<td>4</td>
<td>NA</td>
<td>16</td>
<td>8</td>
<td>21</td>
</tr>
<tr>
<td>Ending Month</td>
<td>7</td>
<td>15</td>
<td>23</td>
<td>15</td>
<td>NA</td>
<td>NA</td>
<td>7</td>
<td>36</td>
<td>NA</td>
<td>36</td>
<td>20</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 3: Optimal solution for allowing preemption

Entry with “NA” in the field denotes the project never starts within 36 periods. According to 3, there are 9 projects operating during the period. CalInvestor can generate a net revenue of 105.6 millions dollars within 3 years, at maximum. Compared with the original linear program, this optimal solution proves the assumption that preemption will generate greater net profit. A detailed project portfolio management schedule is plotted below.

Figure 2: Optimal project portfolio for allowing preemption

4.3 Exploration: Budget Variation

Another important factor limiting the project management is the budget constraint, 5 millions per month. If the budget limit increases, it is possible that more projects can be completed so that the total profit will increase. By putting more resources into project management, an increase of profit will be seen.

There will be a slight modification to the original model too. Everything but one constraint should stay the same with the model in Section 3. Then the only constraint to be modified is constraint 9. Instead of setting the budget limit to be 5 millions per month, a possibly new upper bound can be
set to 7 millions. The modified constraint is shown below.

\[ \sum_{i=1}^{12} O_{ij} \cdot c_i \leq 7, \forall j \in [1, 36] \]  

(25)

AMPL code for preemption also needs to be modified to reflect the change. The change is commented in the Section 6 Appendix. After running the modified code, the optimal solution is shown in Table 4.

<table>
<thead>
<tr>
<th>Project</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting Month</td>
<td>1</td>
<td>1</td>
<td>8</td>
<td>14</td>
<td>NA</td>
<td>NA</td>
<td>1</td>
<td>22</td>
<td>NA</td>
<td>25</td>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>Ending Month</td>
<td>7</td>
<td>6</td>
<td>17</td>
<td>22</td>
<td>NA</td>
<td>NA</td>
<td>7</td>
<td>33</td>
<td>NA</td>
<td>36</td>
<td>19</td>
<td>31</td>
</tr>
</tbody>
</table>

Table 4: Optimal solution for increasing budget limit

Entry with "NA" in the field denotes the project never starts within 36 periods. According to 4, there are 9 projects operating during the period. CalInvestor can generate a net revenue of 105.6 millions dollars within 3 years, at maximum. Compared with the original linear program, this optimal solution proves the assumption that increasing budget limit will generate greater net profit. A detailed project portfolio management schedule is plotted below.

Figure 3: Optimal project portfolio for increasing budget limit

4.4 Exploration: Budget Variation with Preemption

The last exploration for more efficient project management is the combination of Section 4.2 and 4.3. Allowing preemption and increasing the budget limit may help increase the total profit significantly. The model will be changed according to those two sections. After running the modified code, the optimal solution is shown in Table 5.

Entry with "NA" in the field denotes the project never starts within 36 periods. According to 5, there are 10 projects operating during the period. CalInvestor can generate a net revenue of 106.6 millions dollars within 3 years, at maximum. Compared with the previous three cases, this optimal solution proves the assumption that a combination of preemption and increasing budget limit will generate the highest net profit. A detailed project portfolio management schedule is plotted below.

Figure 4: Optimal project portfolio for increasing budget limit and preemption
### Summary

Previous sections focus on calculating the optimal solution of project management under four different situations. Modification to the original linear program includes preemption, increasing budget limit, and a combination of both. Total profit from all projects can be summarized in the table below.

<table>
<thead>
<tr>
<th></th>
<th>5 millions budget</th>
<th>7 millions budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preemption disallowed</td>
<td>$104.2</td>
<td>$105.6</td>
</tr>
<tr>
<td>Preemption allowed</td>
<td>$105.6</td>
<td>$106.6</td>
</tr>
</tbody>
</table>

Table 6: Total net profit summary for all possible improvement

From Table 6, it seems pretty obvious that a combination of preemption and increasing budget limit will increase the total net profit. However, comparing profit among all possible scenarios is not sufficient. Net profit is an absolute value which is weak to compare with, especially under different total costs. A proper approach is comparing the relevant profit, total net profit relevant to total cost. The ratio of total net profit to total cost is called expected rate of return, capturing the amount one would anticipate receiving on a certain amount of investment. The total net profit is the difference between total revenue and total cost. Therefore, using expected rate of return is critical to analyze the performance of those project portfolios.

Total revenue can be easily calculated, which is the sum of profit of all running projects within the 36 periods. In mathematical notation, \( \sum_{j=1}^{36} S_{ij} \cdot p_i \) denotes the profit for individual project \( i \). This is also part of the objective function defined in Section 3.2. Summing up the revenue for all 12 projects can achieve the total revenue. Similarly, total cost can be calculated by summing up the
cost of each individual project. From the objective function, it is denoted as \( \left( \sum_{j=1}^{36} S_{ij} \right) \cdot c_i \cdot d_i \).

The cost for each individual project \( i \) is the product of its duration and the cost per operating month. Therefore, the expected rate of return can be calculated using the formula below.

\[
\text{Expected Rate of Return} = \frac{\text{Total Net Profit}}{\text{Total Cost}} = \frac{\text{Total Revenue} - \text{Total Cost}}{\text{Total Cost}} = \frac{\text{Total Revenue}}{\text{Total Cost}} - 1 = \frac{\sum_{i=1}^{12} \left( \left( \sum_{j=1}^{36} S_{ij} \right) \cdot p_i \right)}{\sum_{i=1}^{12} \left( \left( \sum_{j=1}^{36} S_{ij} \right) \cdot c_i \cdot d_i \right)} - 1
\]  

(26)

Therefore, total revenue, total cost, and expected rate of return can be easily calculated based on Table 1, 2, 3, 4, and 5. A summary of those calculation under four situations is shown below.

<table>
<thead>
<tr>
<th></th>
<th>5 millions budget</th>
<th>7 millions budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preemption disallowed</td>
<td>$225.0</td>
<td>$248.0</td>
</tr>
<tr>
<td></td>
<td>$120.8</td>
<td>$142.2</td>
</tr>
<tr>
<td></td>
<td>0.8626</td>
<td>0.7416</td>
</tr>
<tr>
<td>Preemption allowed</td>
<td>$248.0</td>
<td>$268.0</td>
</tr>
<tr>
<td></td>
<td>$142.2</td>
<td>$161.4</td>
</tr>
<tr>
<td></td>
<td>0.7416</td>
<td>0.6605</td>
</tr>
</tbody>
</table>

Table 7: Total revenue, cost and expected rate of return summary for all possible improvement

Three numbers in each cell are total revenue, total cost and expected rate of return under a specific improvement strategy. A bigger number of the expected rate of return implies a higher payoff under the same investment budget. If the total cost stays the same, as the total revenue goes up, the expected rate of return also increases. Similarly, if the total revenue stays the same, as the total cost goes up, the expected rate of return decreases. Therefore, in those four scenarios above where total cost and total revenue increase altogether, the change of expected rate of return is then determined by a tradeoff between those two values. Surprisingly, from the figures in Table 7, a combination of preemption and increasing budget limit achieves the lowest expected rate of return. It implies that the increase in total revenue is much more than the increase in total cost, resulted in a decrease in the expected rate of return. Moreover, the best expected rate of return is the original case, without preemption and increasing budget limit. The analysis above on the expected rate of return implies the original linear program is the most investment-efficient strategy, which generates the highest expected rate of return.

5 Recommendations

According to the analysis in Section 4, a combination of both allowing preemption and increasing budget limit will generate the highest net profit, among all possible scenarios. However, after the analysis of the expected rate of return, the original case, without preemption and increasing budget, actually has the highest rate of return. A higher expected rate of return implies the investment is more efficiently, allocating decent resources for investment activities. The difference of the total cost in the original solution and in the modified solution can be used in other financial activities in
the company, which may generate an even higher profit. Therefore, it is suggested to follow project starting and ending schedule according to Table 2 and a detailed diagram in Figure 1 on page 8.

The analysis here only focuses on two easily changeable constraints, budget limit and preemption. There are much more constraints can be slightly modified to achieve possible higher payoff and expected rate of return. A further analysis on various constraints modification may be done in the future. However, solving this question is usually considered as a very hard problem in practice.

6 Appendix

6.1 AMPL Data Input

```AMPL
data;

param project := 12;
param month := 36;

param cost :=
    1 1.1
    2 1.2
    3 1.3
    4 1.4
    5 2.5
    6 3.6
    7 1.7
    8 1.8
    9 1.9
   10 2
   11 1.8
   12 1.9;

param duration :=
    1 7
    2 6
    3 10
    4 9
    5 8
    6 6
    7 7
    8 12
    9 10
   10 12
   11 12
   12 12;
```

6.2 AMPL Code

# the number of the project
param project;
# the month of the project
param month;

# Constraint 1
# if project i starts at month j
var start {1..project, 1..month} binary;
# if project i completely ends at month j
var end {1..project, 1..month} binary;
# if project i is operating at month j
var operating {1..project, 1..month} binary;

param cost {1..project} > 0;
param duration {1..project} > 0;
param profit {1..project} > 0;

maximize Total_Profit: sum {i in 1..project} (sum {j in 1..month} start[i,j] * ( profit[i] - cost[i] * duration[i] ));

subject to
# Constraint 2
One_Start {i in 1..project}:
sum {j in 1..month} start[i, j] <= 1;

# Constraint 3
One_End {i in 1..project}:
sum {j in 1..month} end[i, j] <= 1;
# Constraint 4
Operating_Is_Duration {i in 1..project}:
    sum {j in 1..month} operating[i, j] = duration[i] *
    sum {j in 1..month} start[i, j];

# Constraint 5
Complete_or_NeverStart {i in 1..project}:
    sum {j in 1..month} start[i, j] = sum {j in 1..month} end[i, j];

# Constraint 6
End_to_Start {i in 1..project}:
    sum {j in 1..month} end[i, j] * j -
    sum {j in 1..month} start[i, j] * j =
    (duration[i] - 1) * sum {j in 1..month} start[i, j];

# Under preemption situation, the equal sign in constraint 6 needs
# to be changed to greater than or equal to. The following is the
# modified constraint 6, which is listed as constraint 24 in the
# report.
# End_to_Start {i in 1..project}:
#    sum {j in 1..month} end[i, j] * j -
#    sum {j in 1..month} start[i, j] * j >=
#    (duration[i] - 1) * sum {j in 1..month} start[i, j];

# Constraint 7
Limit_Operation_Start {i in 1..project, k in 1..month}:
    sum {j in 1..k} start[i, j] >= operating[i, k];

# Constraint 8
Limit_Operation_End {i in 1..project, k in 1..month}:
    sum {j in k..36} end[i, j] >= operating[i, k];

# Constraint 9
Budget_Limit {j in 1..month}:
    sum {i in 1..project} operating[i, j] * cost[i] <= 5;
# Under increasing budget limit situation, the value 5 in constraint
# 9 needs to be changed to 7. The following is the modified constraint
# 9, which is listed as constraint 25 in the report.
# Budget_Limit {j in 1..month}:
#    sum {i in 1..project} operating[i, j] * cost[i] <= 7;

# Constraint 10
Project_Start_Limit {j in 1..month}:
    sum {i in 1..project} start[i, j] <= 3;

# Constraint 11
Three_b4_Nine {k in 1..month-1}:
    sum {j in 1..k} end[3, j] >= sum {j in 1..k+1} start[9, j];
# Constraint 12
One_b4_Ten \{k \text{ in } 1..\text{month}-1\}:
\[\sum_{j \text{ in } 1..k} \text{end}[1, j] \geq \sum_{j \text{ in } 1..k+1} \text{start}[10, j] ;\]

# Constraint 13
Two_b4_Ten \{k \text{ in } 1..\text{month}-1\}:
\[\sum_{j \text{ in } 1..k} \text{end}[2, j] \geq \sum_{j \text{ in } 1..k+1} \text{start}[10, j] ;\]

# Constraint 14
One_b4_Eleven \{k \text{ in } 1..\text{month}-1\}:
\[\sum_{j \text{ in } 1..k} \text{end}[1, j] \geq \sum_{j \text{ in } 1..k+1} \text{start}[11, j] ;\]

# Constraint 15
Two_b4_Twelve \{k \text{ in } 1..\text{month}-1\}:
\[\sum_{j \text{ in } 1..k} \text{end}[2, j] \geq \sum_{j \text{ in } 1..k+1} \text{start}[12, j] ;\]

# Constraint 16
Six_or_Seven_or_Eight_b4_Eleven \{k \text{ in } 1..\text{month}-1\}:
\[\sum_{j \text{ in } 1..k} \text{end}[6, j] + \sum_{j \text{ in } 1..k} \text{end}[7, j] + \sum_{j \text{ in } 1..k} \text{end}[8, j] \geq \sum_{j \text{ in } 1..k+1} \text{start}[11, j] ;\]

# Constraint 17
Four_or_Five_b4_Ten \{k \text{ in } 1..\text{month}-1\}:
\[\sum_{j \text{ in } 1..k} \text{end}[4, j] + \sum_{j \text{ in } 1..k} \text{end}[5, j] \geq \sum_{j \text{ in } 1..k+1} \text{start}[10, j] ;\]

# Constraint 18
Nine_or_Eleven_b4_Twelve \{k \text{ in } 1..\text{month}-1\}:
\[\sum_{j \text{ in } 1..k} \text{end}[9, j] + \sum_{j \text{ in } 1..k} \text{end}[11, j] \geq \sum_{j \text{ in } 1..k+1} \text{start}[12, j] ;\]

# Constraint 19
One_Three_Eight_no_Overlap \{j \text{ in } 1..\text{month}\}:
\[\text{operating}[1, j] + \text{operating}[3, j] + \text{operating}[8, j] \leq 1;\]

# Constraint 20
Two_Four_no_Overlap \{j \text{ in } 1..\text{month}\}:
\[\text{operating}[2, j] + \text{operating}[4, j] \leq 1;\]

# Constraint 21
Five_Six_no_Overlap \{j \text{ in } 1..\text{month}\}:
\[\text{operating}[5, j] + \text{operating}[6, j] \leq 1;\]

# Constraint 22
Five_1month_b4_Six \{j \text{ in } 1..\text{month}-1\}:
\[\text{operating}[5, j] + \text{operating}[6, j+1] \leq 1;\]

# Constraint 23
Five_1month_after_Six \{ j \text{ in } 2..\text{month}\}:
\begin{align*}
\text{operating}[5, j] + \text{operating}[6, j - 1] & \leq 1;
\end{align*}

List of Figures

1. Optimal project portfolio for the original linear program ........................................... 8
2. Optimal project portfolio for allowing preemption ......................................................... 9
3. Optimal project portfolio for increasing budget limit ................................................. 10
4. Optimal project portfolio for a combination of both changes .................................... 11

List of Tables

1. Operational cost, duration, and total profit for each project ........................................ 2
2. Optimal solution for the original linear program ......................................................... 8
3. Optimal solution for allowing preemption ................................................................. 9
4. Optimal solution for increasing budget limit ............................................................. 10
5. Optimal solution for a combination of both changes .................................................. 11
6. Total net profit summary for all possible improvement ............................................. 11
7. Total revenue, cost and expected rate of return summary for all possible improvement 12